

MONOGRAFIE, STUDIA, ROZPRAWY

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**Antonín Víteček, Leszek Cedro
Radim Farana, Miluše Vítečková**

**THE FUNDAMENTALS
OF MATHEMATICAL MODELLING**



Politechnika Świętokrzyska

Kielce 2018

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PREFACE

This book ‘The fundamentals of mathematical modelling’ is targeted at students of the Faculty of Mechatronics and Mechanical Engineering of the Kielce University of Technology. It provides basic information on the mathematical modelling of the dynamic systems that students may come across during their time at University.

The book consists of six chapters, two annexes and a list of references.

Chapter One is an introduction to and an overview of systems theory. Chapter Two describes the most common mathematical models of dynamic systems. Chapter Three provides linear mathematical models of dynamic systems. Chapter Four discusses the linearization of non-linear mathematical models. Chapter Five deals with the classification of linear dynamic elements. Chapter Six explains how to build mathematical models of different dynamic systems. It focuses on the mathematical modelling of mechanical and electrical systems. The next two chapters include tables of Laplace transforms and the basic physical quantities.

The problems discussed in this book are illustrated with numerous examples and their solutions.

Since the book provides only basic information, there is no in-text citation. The reference list is provided at the end.

For students wishing to extend their knowledge of the modelling of dynamic systems, the authors recommend the following publications:

Kulakowski, B.T., Gardner, J.F., Shearer, J.L.: *Dynamic Modeling and Control of Engineering Systems. Third Edition*. Cambridge University Press, Cambridge 2008, ISBN 978-0-521-86435-0

Mikleš, J., Fikar, M.: *Process Modelling, Identification, and Control*. Springer-Verlag Berlin 2007, ISBN 978-3-540-71969-4

Ogata, K.: *System Dynamics*. Fourth Edition. Pearson Prentice-Hall, Upper Saddle River 2004, ISBN 0-13-1422462-9

Palm III, W.J.: *System Dynamics. Second Edition*. McGraw-Hill, Boston 2010, ISBN 978-0-07-352927-1

To, Cho W.S.: *Introduction to Dynamics and Control in Mechanical Systems*. ASME Press and John Wiley and Sons, Southern Gate 2016, ISBN 9781-1-07-189349920

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