environment

ANATOLIY PAVLENKO
Poltava National Technical Yuri Kondratyuk University (Ukraine)
e-mail: am.pavlenko@i.ua

THE TEMPERATURE DISTRIBUTION OF THE MATERIALS IN THE CONVECTIVE HEAT TRANSFER

Abstract
A new method for solving the equations of thermal conductivity materials with specific properties. This method is useful for an approximate estimate of the temperature field of materials.

Keywords: heat equation, method for solving

1. Introduction

With the intensification of heat transfer processes, a significant increase in operating temperatures and increased requirements for precision thermotechnical calculations gradually and increasingly become apparent shortcomings well-developed theory based on linear boundary value problems of heat conduction (BPHC). Therefore, in the last few decades, increased attention is paid to the non-linear mathematical modeling (MM) thermal processes.

Due to the mathematical difficulties exact solutions of nonlinear BPHC obtained only for some special cases. Generally, for this purpose, various approximate methods, which can be divided into two fundamentally different groups. The first section includes well known (eg [1-3]) exact methods for solving linear BPHC. But the use of these methods must be preceded by the corresponding output linearization of nonlinear problems. The second group includes those approximation methods that allow directly solve nonlinear BPHC without prior linearization. This is primarily numerical and analogue methods and approximate analytical, integrated, variation, disturbance (small parameter) and others. (See., Eg, [4]). These include the method of equivalent sources (MES) [3, 4].

As for the research of thermal processes in the bodies of functionally dependent TFH (non-linearity of the first kind), they are related to the difficulties caused by not only non-linear equation, but the fact that laws change TFH not reliably identified. Of course dependence of the thermal conductivity $\lambda(T)$ and volumetric specific heats $C(T)$ approximated (See., Eg, [4,]) by polynomials

$$\lambda(T) = \lambda_0 + \sum_{i=1}^{n} \delta_{\lambda i} T^i \; ; \; C(T) = C_0 + \sum_{i=1}^{n} \delta_{Ci} T^i \qquad (1)$$

For many materials coefficients T^i are so small that $i \geq 2$ members of can be negligible that to some extent consistent with modern concepts of the mechanism of thermal conductivity as a superposition of streams of photons and electrons, where the fate of metals in the past for a specific temperature range is dominant.

So the future will consider nonlinear (quasi-linear) differential equations containing linearly dependent on temperature TFH that after going to the relevant dimensionless quantities take the form of:

$$\begin{aligned} &\lambda(\theta) = \lambda_0 \cdot \bar{\lambda}(\theta); \\ &\bar{\lambda}(\theta) = 1 + \varepsilon_\lambda \theta; \\ &\varepsilon_\lambda = \delta_\lambda (T_C - T_0)/\lambda_0; \\ &C(\theta) = C_0 \cdot \bar{C}(\theta); \\ &\bar{C}(\theta) = 1 + \varepsilon_C \theta; \\ &\varepsilon_C = \delta_C (T_C - T_0)/C_0 \end{aligned} \qquad (2)$$

Introduced here coefficients ε_λ, ε_C will henceforth be called nonlinearity parameters of the 1st kind.

2. Statement of the problem

If we restrict ourselves to the linear case TFH (2), the temperature state (TS) unlimited plate in convective heat transfer solutions of the BPHC determined:

$$\frac{\partial}{\partial\xi}\left[\left(1+\varepsilon_\lambda\theta\right)\frac{\partial\theta}{\partial\xi}\right]=\left(1+\varepsilon_C\theta\right)\frac{\partial\theta}{\partial\tau};\qquad \theta(\xi,0)=0 \tag{3}$$

$$\left(1+\varepsilon_\lambda\theta_\Pi\right)\frac{\partial\theta}{\partial\xi}\bigg|_{\xi=1}=Bi\left[\theta_C-\theta_\Pi(\tau)\right];\qquad \frac{\partial\theta}{\partial\xi}\bigg|_{\xi=0}=0 \tag{4}$$

where

$$\theta(\xi,\tau)=\frac{T(\xi,\tau)-T_0}{T_C-T_0};\quad \xi=\frac{x}{H};\quad \tau=\frac{a_0t}{H^2};\quad Bi=\frac{\alpha_K H}{\lambda_0}; \tag{5}$$

$T(\xi,\tau)$ – the body temperature, K; T_0 – its initial value, K; T_C – temperature of the heating medium, K; x – coordinate (starting from the center section), m; $2H$ – plate thickness, m; t – time, hour; $a_0=\lambda_0/C_0$ – thermal diffusivity coefficient in $T=T_0$, m^2/hour; α_K – convective heat transfer coefficient, W/(m^{2}K).

To solve this BPHC (3), (3) apply the method of equivalent sources (MES) in the well-known theory of heat near-boundary layer (engineering model of thermal conductivity), which examines the process of conductive heat transfer in two stages: initial heating (inertial phase $0\le\tau\le\tau_0$) and heating for the whole volume (ordered phase $\tau\ge\tau_0$).

3. Solution of the problem

The first (inertial) stage ($0\le\tau\le\tau_0$; $\beta(\tau)\le\xi\le1$) solving equations MES take the form:

$$\frac{\partial}{\partial\xi}\left[\left(1+\varepsilon_\lambda\theta_1\right)\frac{\partial\theta_1}{\partial\xi}\right]=f_1(\tau) \tag{6}$$

where "source equivalent" $f_1(\tau)$ is defined by the integral

$$f_1(\tau)=\frac{1}{1-\beta(\tau)}\int_{\beta(\tau)}^{1}\left[\left(1+\varepsilon_C\theta_1\right)\frac{\partial\theta_1}{\partial\tau}\right]d\xi \tag{7}$$

Instead symmetry condition $(4)_2$ introduces coupling conditions of temperature fields is not warmed ($(0\le\xi\le\beta(\tau))$ $\theta_0=0$) and warmed ($\beta(\tau)\le\xi\le1$) $\theta_1(\xi,\tau)$) zones on the border $\xi=\beta(\tau)$ of their distribution

$$\partial\theta_1/\partial\xi\big|_{\xi=\beta(\tau)}=0;\quad \theta_1(\xi,\tau)\big|_{\xi=\beta(\tau)}=0 \tag{8}$$

Integrating solving equations (6) and twice by ξ using the condition (8), we obtain

$$\theta_1(\xi,\tau)=\frac{1}{\varepsilon_\lambda}\left\{\sqrt{1+\varepsilon_\lambda f_1(\tau)\left[\xi-\beta(\tau)\right]^2}-1\right\} \tag{9}$$

After substitution function (9) in the boundary condition (BC) $(5)_1$ we obtain

$$f_1(\tau)=\frac{Bi\left[2\left(1+\varepsilon_\lambda\right)+Bil(\tau)\right]}{2\varepsilon_\lambda l(\tau)}\times\left\{1-\sqrt{1-\frac{4\varepsilon_\lambda\left(2+\varepsilon_\lambda\right)}{\left[2\left(1+\varepsilon_\lambda\right)+Bil(\tau)\right]^2}}\right\} \tag{10}$$

Expanding radical (10) in power series and retaining the first two of its members have

$$f_1(\tau)\approx\frac{\left(2+\varepsilon_\lambda\right)Bi}{\left[2\left(H\varepsilon_\lambda\right)+Bil(\tau)\right]l(\tau)} \tag{11}$$

Thus the inertial phase approximate solution of problem (3), (4) takes the form

$$\theta_1(\xi,\tau)=\frac{1}{\varepsilon_\lambda}\left\{\sqrt{1+\frac{\varepsilon_\lambda\left(2+\varepsilon_\lambda\right)Bi\left[\xi-\beta(\tau)\right]^2}{\left[2\left(1+\varepsilon_\lambda\right)+Bil(\tau)\right]l(\tau)}}-1\right\} \tag{12}$$

Position Front warming $l(\tau)=1-\beta(\tau)$ in principle determined integral condition (7). However, this procedure results in inappropriate complex calculations. Due to the rapidity inertial stage we assume TFH change during this period was insignificant and determined $l(\tau)$ by the solution of the linear problem

$$l^2+4l/Bi-\frac{8}{Bi^2}\ln\left(1+Bil/2\right)=12\tau \tag{13}$$

$$\tau_0=\left[1+4/Bi-\frac{8}{Bi^2}\ln\left(1+Bi/2\right)\right]\Big/12 \tag{14}$$

or simplified

$$l(\tau)=\sqrt{12\left(3.25+Bi\right)\tau/\left(6.5+Bi\right)};\qquad \tau_0=\frac{6.5+Bi}{12\left(3.25+Bi\right)} \tag{15}$$

At low ($Bi\ll3.25$) and high ($Bi>6.5$) respectively have

environment

$$
\begin{gathered}
l(\tau)=\sqrt{6\tau};\\
\tau_0=1/6;\\
l(\tau)=\sqrt{12\tau};\\
\tau_0=1/12
\end{gathered}
\tag{16}
$$

At the time τ_0 of completion warming $l(\tau_0) = 1$, $\beta(\tau_0) = 0$

$$
\theta_1(\xi,\tau_0)=\theta_1^0(\xi)=
=\frac{1}{\varepsilon_\lambda}\left[\sqrt{1+\frac{\varepsilon_\lambda(2+\varepsilon_\lambda)Bi}{2(1+\varepsilon_\lambda)+Bi}\xi^2}-1\right]
\tag{17}
$$

$$
\begin{gathered}
\theta_1^0(\xi)\Big|_{\xi=1}=\theta_{1\Pi}^0=\frac{1}{\varepsilon_\lambda}\left[\sqrt{1+\frac{\varepsilon_\lambda(2+\varepsilon_\lambda)Bi}{2(1+\varepsilon_\lambda)+Bi}}-1\right];\\
\theta_1^0(\xi)\Big|_{\xi=0}=\theta_{1C}^0=0
\end{gathered}
\tag{18}
$$

The second (ordered) phase ($\tau \ge \tau_0$; $0 \le \xi \le 1$) Solving equations similar to (6):

$$
\frac{\partial}{\partial\xi}\left[(1+\varepsilon_\lambda\theta_2)\frac{\partial\theta_2}{\partial\xi}\right]=f_2(\tau)
\tag{19}
$$

where

$$
\begin{gathered}
f_2(\tau)=\int_0^1(1+\varepsilon_c\theta_2)\frac{\partial\theta_2}{\partial\tau}d\xi=\\
=\frac{1}{2\varepsilon_c}\frac{d}{d\tau}\int_0^1(1+\varepsilon_\lambda\theta_2)d\xi
\end{gathered}
\tag{20}
$$

Integrating equation (19) and using the BC (4), we find

$$
\begin{gathered}
\theta_2(\xi,\tau)=\frac{1}{\xi\lambda}\left\{\sqrt{1+\psi(\tau)}-1\right\}\\
\psi(\tau)=2\varepsilon_\lambda\left[1-\frac{f_2(\tau)}{2}\left(\frac{2+Bi}{Bi}-\xi^2\right)\right]+\\
+\varepsilon_\lambda^2\left[1-\frac{f_2(\tau)}{Bi}\right]^2
\end{gathered}
\tag{21}
$$

Substituting the function (21) in the condition (20) we obtain the expression

$$
\begin{gathered}
2\varepsilon_c f_2(\tau)=\frac{d}{d\tau}\int_0^1\left[\left(1-\frac{\varepsilon_c}{\varepsilon_\lambda}\right)^2+2\left(1-\frac{\varepsilon_c}{\varepsilon_\lambda}\right)\times\right.\\
\left.\times\frac{\varepsilon_c}{\varepsilon_\lambda}\theta_2(\xi,\tau)+\frac{\varepsilon_c^2}{\varepsilon_\lambda^2}\theta_2^2(\xi,\tau)\right]d\xi
\end{gathered}
\tag{22}
$$

In order to simplify some free from radical and integral (22), which function $\theta_2(\xi, \tau)$ (21) in the second term (22) replace the first two members of its power series, i.e.

$$
\begin{gathered}
\theta_2(\xi,\tau)\approx 1+\\
+\varepsilon_{\tilde N}\left[1+\frac{f_2(\tau)}{2}\left(\frac{2+Bi}{Bi}-\xi^2\right)+\frac{\varepsilon_\lambda^2}{2}\left(1+\frac{f_2(\tau)}{Bi}\right)^2\right]
\end{gathered}
\tag{23}
$$

In this case, expression (22) takes a simpler form. Keeping the right side of (22) members only dependent on the time τ, we arrive at the differential equation

$$
\left[\frac{\varepsilon_\lambda}{Bi}-\frac{3(1+\varepsilon_\lambda)+Bi}{3f_2(\tau)}\right]df_2=Bi\,d\tau
\tag{24}
$$

integral which is defined transcendent expression:

$$
\begin{gathered}
\frac{\varepsilon_\lambda}{Bi}\left[f_2(\tau)-f_2^0\right]-\left[(1-\varepsilon_\lambda)+\frac{Bi}{3}\right]\times\\
\times ln\frac{f_2}{f_2^0}=Bi(\tau-\tau_0)
\end{gathered}
\tag{25}
$$

In order to receive functions $f_2(\tau)$ in an explicit form shall spread out $\ln f_{12}(\tau)$ in power series

$$
\ln f_2(\tau)=\sum_{n=1}^{\infty}(-1)^{n-1}\left[f_2(\tau)-1\right]^n/n
\tag{26}
$$

and limited to the first two of its members ($n = 1$), we find

$$
f_2(\tau)\approx 1+\ln f_2(\tau)
\tag{27}
$$

Then equation (25) after substituting in the first term of the function (27) leads to the expression:

$$
f_2(\tau)=f_2^0\exp\left\{-\frac{3Bi^2(\tau-\tau_0)}{Bi\left[3(1+\varepsilon_\lambda)+Bi\right]-3\varepsilon_\lambda}\right\}
\tag{28}
$$

Used simplification (23) and (27) is quite legitimate because it relates to members, usually containing the nearest multiple small parameters ε_λ and ε_C nonlinearity, ie secondary member, and do not touch the main members of the inherent solution of the corresponding linear problem ($\varepsilon_\lambda = \varepsilon_C = 0$).

Introducing the notation

$$
\mu=\frac{3Bi}{3(1+\varepsilon_\lambda)+Bi-3\varepsilon_\lambda/Bi}
\tag{29}
$$

we present the solution (21) function

$$
\begin{gathered}
\theta_2(\xi,\tau)=\frac{1}{\xi\lambda}\left\{\sqrt{1+\psi(\xi,\tau)+\psi(\tau)}-1\right\}\\
\psi(\xi,\tau)=2\varepsilon_\lambda\left\{1-\frac{f_2^0}{2}\left(\frac{2+Bi}{Bi}-\xi^2\right)exp\left[-\mu(\tau-\tau_0)\right]\right\}\\
\psi(\tau)=\varepsilon_\lambda^2 1-(f_2^0)/Biexp[-\mu(\tau-\tau_0)]^2
\end{gathered}
\tag{30}
$$

Sustainable integration f_2^0 of find conditions

$$\theta_2\left(\xi,\tau\right)\Big|_{\substack{\xi=0 \\ \tau=\tau_0}} = \theta_{1C}^0 = 0 \quad (18)$$

$$f_2^0 = \frac{\left[2\left(1+\varepsilon_\lambda\right)+Bi\right]Bi}{2\varepsilon_\lambda}\times \\ \times\left\{1-\sqrt{1-\frac{4\varepsilon_\lambda\left(2+\varepsilon_\lambda\right)}{\left[2\left(1+\varepsilon_\lambda\right)+Bi\right]^2}}\right\} \quad (31)$$

This ends solution of the nonlinear heat conduction problem.

References

[1] Carslaw G., Eger D.: *Thermal conductivity of solids*. – M: Mir, 1964, 487 p.
[2] Lykov A.V.: *The theory of heat conduction*. – M: Higher School, 1967, 599 p.
[3] Tikhonov A.N., Samarskii A.A.: *Equations of mathematical physics*. – M: Nauka, 1972, 735 p.
[4] Belyaev N.M., Ryadno A.A. *Methods of the theory of heat conduction, in 2 parts*. – M: Higher School, 1982, 631 p.